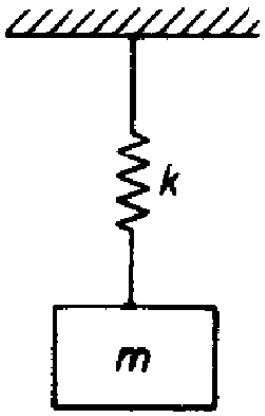
Problem 2.4

An unknown mass of m kg attached to the end of an unknown spring k has a natural frequency of 94 cpm. When a 0.453-kg mass is added to m, the natural frequency is lowered to 76.7 cpm. Determine the unknown mass m and the spring constant k N/m.

Solution

Below is the system with unknown spring stiffness k and unknown mass m.



The natural frequency for this system is

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 94\ \frac{\text{cycles}}{\cancel{\text{minute}}} \times \frac{\cancel{1\ \text{minute}}}{60\ \text{seconds}} = \frac{47}{30}\ \frac{\text{cycles}}{\text{second}}. \tag{1}$$

We have one equation with two unknowns, k and m. A second equation is needed to solve for both of them. When the 0.453-kg mass is added, the natural frequency becomes

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{m+0.453}} = 76.7\ \frac{\text{cycles}}{\cancel{\text{minute}}} \times \frac{\cancel{1\ \text{minute}}}{60\ \text{seconds}} = \frac{767}{600}\ \frac{\text{cycles}}{\text{second}}. \tag{2}$$

Now solve the system of equations.

$$\begin{cases} \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} = \dfrac{47}{30} \\ \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m+0.453}} = \dfrac{767}{600} \end{cases}$$

$$\begin{cases} \sqrt{\dfrac{k}{m}} = \dfrac{47\pi}{15} \\ \sqrt{\dfrac{k}{m+0.453}} = \dfrac{767\pi}{300} \end{cases}$$

$$\begin{cases} \sqrt{k} = \dfrac{47\pi}{15}\sqrt{m} \\ \sqrt{k} = \dfrac{767\pi}{300}\sqrt{m+0.453} \end{cases}$$

Since the right side of each equation is equal to $\sqrt{k}$, they must be equal to each other.

$$\frac{47\pi}{15}\sqrt{m} = \frac{767\pi}{300}\sqrt{m+0.453}$$
$$\sqrt{m} = \frac{767}{940}\sqrt{m+0.453}$$
$$m = \frac{588\,289}{883\,600}(m+0.453)$$
$$m\left(1-\frac{588\,289}{883\,600}\right) = \frac{588\,289}{883\,600}\times 0.453$$
$$\boxed{m = \frac{\frac{588\,289}{883\,600}\times 0.453}{1-\frac{588\,289}{883\,600}} \approx 0.902 \text{ kg}}$$

Use one of the formulas from the system,

$$\sqrt{k} = \frac{47\pi}{15}\sqrt{m},$$

to solve for k.

$$k = \frac{2209\pi^2}{225}m$$

Therefore,

$$\boxed{k = \frac{2209\pi^2}{225}\frac{\frac{588\,289}{883\,600}\times 0.453}{1-\frac{588\,289}{883\,600}} \approx 87.4 \frac{\text{N}}{\text{m}}.}$$